Trinity College

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	101	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section One: Calculator-free **35% (54 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Let the displacement vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be $(11, -4)$, $(5, 14)$ and $(8, m)$ respectively, where m is a constant.

(a) Determine the vector $3\mathbf{a} + 2\mathbf{b}$. (2 marks)

(b) Given that $(\mathbf{a} + \mathbf{b} + k\mathbf{c}) = 0$, detemine the values of k and m. (3 marks)

Question 2 **(6 marks)**

Relative to the origin O, points A and B have position vectors $-3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{i} - 4\mathbf{j}$ respectively.

(a) Determine the unit vector $\hat{\mathbf{c}}$, where $\mathbf{c} = \overrightarrow{AB}$. (3 marks)

(b) Vector $\mathbf{d}$ has magnitude $3\sqrt{5}$, is parallel to $\mathbf{c}$ and in the opposite direction. Determine $\mathbf{d}$. (3 marks)

See next page

Question 3 **(8 marks)**

Consider the following statement about a simple (no edges that cross) polygon:

If it has an interior angle sum of 360°, then it is a square.

(a) Use a counter-example to explain why the statement is false. (2 marks)

(b) Write the converse statement and state whether it is always, sometimes or never true. (2 marks)

(c) Write the inverse statement and state whether it is always, sometimes or never true. (2 marks)

(d) Write the contrapositive statement and state whether it is always, sometimes or never true. (2 marks)

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Question 4 **(6 marks)**

(a) Determine the value of the constant n, given that the vectors $12\mathbf{i} + n\mathbf{j}$ and $5\mathbf{i} - 8\mathbf{j}$ are perpendicular. (2 marks)

(b) The vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $|\mathbf{a}| = 18, |\mathbf{b}| = 12$ and $\mathbf{a} \cdot \mathbf{b} = -3$. Evaluate

(i) $-2\mathbf{a} \cdot 3\mathbf{b}$. (1 mark)

(ii) $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a})$. (3 marks)

Question 5 **(6 marks)**

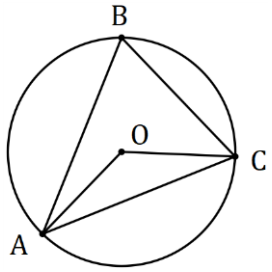
A drone leaves point P and travels 74 m on bearing of $290°$ to Q, then 40 m on bearing $020°$ to R and finally 114 m on bearing $110°$ to S.

(a) Sketch a neat diagram to show the path of the drone. (2 marks)

(b) The drone is to return directly from S to P. Determine the distance it must fly and on what bearing. (4 marks)

Question 6 **(7 marks)**

(a) In the diagram below, the vertices of triangle ABC lie on a circle with centre O. Given that $\angle ABC = 63°$, determine the values of $\angle AOC$ and $\angle OAC$. (2 marks)

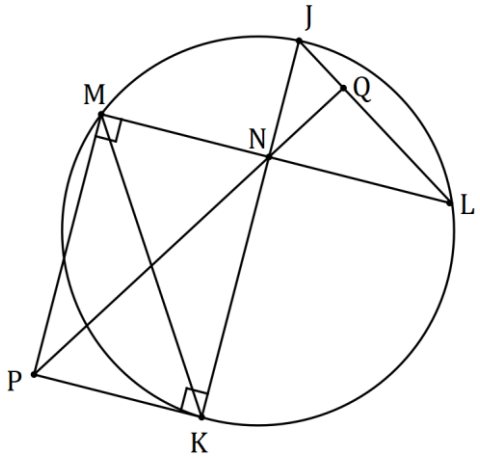


(b) Prove, assuming only basic axioms and properties of triangles, that the size of the angle at the centre subtended by an arc of a circle is twice the size of the angle at the circumference subtended by the same arc. (5 marks)

See next page

Question 7 **(7 marks)**

In the diagram below, two chords of a circle, JK and LM, intersect at N. PM is perpendicular to LM at M and PK is perpendicular to JK at K. The line PN intersects chord JL at Q.



(a) Explain why $PMNK$ is a cyclic quadrilateral. (1 mark)

(b) Prove that $\angle MPN = \angle JLM$. (3 marks)

(c) Prove that PQ is perpendicular to JL. (3 marks)

Question 8 **(9 marks)**

(a) Evaluate ${}^{25}P_{19} \div {}^{23}P_{20}$. (3 marks)

(b) Express $8! + 7! + 6!$ in the form $a^2b!$, where a and b are positive integers. (3 marks)

(c) Show that for $n\ an\ integer, n \geq 1$, the sum $(n+2)! + (n+1)! + n!$ can always be expressed in the form $a^2b!$ where a and b are positive integers. (3 marks)

End of questions

Supplementary page

Question number: _____